Table 1 Maximum absolute values of the stream function and the location where they occur ($B = H = 2$)

Gr.Pr	ϵ	$\lvert\Psi\rvert_{max}\sqrt{Gr}$	(x, y)
10^4	BA[a]	0.241	(3.22, 3.15)
10^4	0.2	0.234	(3.08, 3.04)
10^4	1	0.220	(2.73, 2.53)
10^4	2	0.183	(2.47, 2.27)
10^5	BA	0.148	(3.08, 3.39)
10^5	1	0.125	(2.97, 3.30)
10^5	2	0.114	(2.73, 3.02)

[a]BA = Boussinesq approximation.

increasing values of ϵ. When an entirely cold side wall ($B = 5$, $H = 0$) is considered (results not shown), little change is seen in the field structure except for local deviations in the region near the cold wall, compared to cases with the shorter cooling element ($B = H = 2$).

Table 1 shows maximum absolute values of the stream function and their location over the entire range of the parameters investigated (Gr.Pr and ϵ). The intensity of the flow indicated by $\lvert\Psi\rvert_{max}$ is seen to decrease as ϵ increases for any given Gr.Pr. The shift of the location of the vortex center toward the left lower corner of the cavity accompanies this change. These values also indicate that only a slight increase in the intensity of the flow occurs even when the size of the cooling element is more than doubled.

Conclusions

For the range of the density difference parameter ϵ (≤ 2.0) investigated, the intensity of convection was found to be reduced as ϵ increased. This change appears as the thermal and velocity boundary layers thicken on the enclosure surfaces and the maximum values of the stream function decreases and shift their location toward the cavity center.

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Thermocapillary Migration of a Large Gas Slug in a Tube

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Nomenclature

- A = temperature gradient at the tube wall
- C_i = constants
- K = nondimensional pressure gradient, Eq. (2)
- L = length of the vapor slug
- Ma = Marangoni number $V_\infty R_2/\alpha$
- P, p = pressure, dimensional and nondimensional
- R_1 = radius of the slug
- R_2 = radius of the tube
- t = time
- T = temperature
- V, v = axial velocity, dimensional and nondimensional
- V_∞ = terminal velocity of the slug
- Z, z = axial coordinate, dimensional and nondimensional
- α = thermal diffusivity
- μ = dynamic viscosity of the liquid
- σ = surface tension
- σ_T = $d\sigma/dT$
- ϕ = dimensionless temperature

Introduction

IN a zero gravity environment, the liquid motion and migration velocities of bubbles and drops in the absence of imposed forced flow will principally be determined by the surface-tension gradient at the interface. The surface-tension gradient may be induced either by a temperature or concentration gradient. The steady thermocapillary motion of bubbles placed in an infinite medium with a linear temperature gradient has been investigated extensively. Young et al.[1] obtained an expression for the terminal velocity of the migration of a bubble, neglecting inertia and the convection of energy. The subsequent studies have relaxed some of the assumptions in Ref. 1. A comprehensive review of the thermocapillary migration of bubbles is given by Thompson.[2] Balasubramaniam and Chai[3] recently reconsidered the problem and obtained an exact solution of the thermocapillary motion of droplets for small Marangoni numbers, but arbitrary Reynolds numbers.

Received Aug. 7, 1987; revision received Jan. 25, 1988.

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Because of its importance in materials processing under microgravity and even under normal gravity, the aforementioned studies are basically concerned with the thermocapillary migration of small bubbles. However, under microgravity condition, the thermocapillary phenomena may be important for a wide variety of fluid dynamic problems. Bubbles in a tank or tube containing liquid will not necessarily rise to the top; rather, they will migrate to the hotter portion of the fluid. The purpose of this study is to present an analysis of the thermocapillary flow around a gas slug in a tube filled with liquid and subjected to a linear temperature distribution along the tube wall.

Analysis

Consider a long tube of radius R_2 as shown in Fig. 1, filled with liquid, containing a gas slug of cylindrical shape of radius R_1 and length L. The tube wall and liquid away from the bubble are subjected to a linear temperature gradient in the axial direction. The surface-tension gradient at the bubble surface, caused by the temperature variation, exerts a shear stress on the liquid, pushing the liquid from the hot to cold side. The reaction that the liquid exerts on the bubble will cause it to move to the hot side. After an initial transient, the bubble will migrate toward the hotter fluid with a constant terminal velocity. The objective of this study is to determine the terminal velocity under some simplifying assumptions.

In this analysis, it is assumed that the thermocapillary force is the only driving force. Gravity is zero and there is no imposed flow. No phase change occurs and there is no mass transfer. It is assumed that the liquid wets the solid tube wall, and hence there is a liquid film between the tube wall and bubble. The viscosity and thermal conductivity of the gas are neglected compared to that of the liquid. The deformation of the bubble and the interfacial instability are ignored. The bubble is cylindrical in shape, with $R_1/L \ll 1$. Also, the thickness of the annulus is small, i.e., $(R_2 - R_1)/R_1 \ll 1$. As a consequence of all the assumptions made, we suggest that the flow within the annulus will be fully developed. This will not be true near the ends of the gas slug. However, if the length of the gas slug is sufficiently large compared to its radius, the error incurred in the result for the terminal velocity may be negligible.

Governing Equations and Solution

We select a coordinate system on the bubble with the origin at its center of mass. In this system, the bubble is stationary and the tube along with the liquid approaches the bubble with the terminal velocity V_∞. The velocity field will be steady since V_∞ is constant. However, the temperature field will not be steady because, as time increases, the bubble is subjected to increasing temperatures. The conservation equations for the case under consideration are written in the following nondimensional form, with the nondimensional variables being defined as

$$r = R/R_2, \quad z = Z/R_2, \quad v = V/V_\infty, \quad p = P/(\mu V_\infty/R_2),$$

$$\phi = (T - AV_\infty t)/(AR_2)$$

$$\partial v/\partial z = 0 \tag{1}$$

$$(1/r)\,\partial(r\partial v/\partial r)/\partial r = \partial p/\partial z = -K \tag{2}$$

$$1 + v\partial\phi/\partial z = (1/\text{Ma})\nabla^2\phi \tag{3}$$

where $\text{Ma} = V_\infty R_2/\alpha$.

The solutions of Eqs. (2) and (3) are

$$v = -Kr^2/4 + C_1 \ln r + C_2 \tag{4}$$

$$\phi = -z + f(r) \tag{5}$$

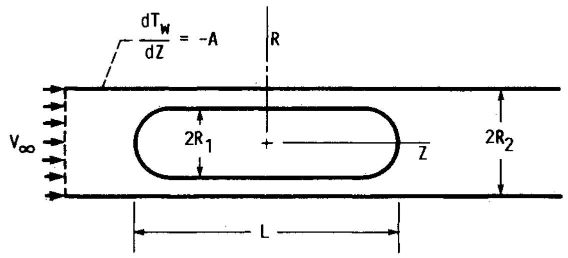

Fig. 1 Coordinate system and physical model. The origin of the coordinate system is at the center of mass of the gas slug.

where

$$f(r) = \text{Ma}\,[Kr^4/64 + r^2/4 - (C_1/4)r^2(\ln r - 1/2) + C_1r^2/8 + C_2r^2/4] + C_3 \ln r + C_4 \tag{6}$$

Equations (4–6) contain six unknowns C_1, C_2, C_3, C_4, K, and V_∞. The boundary conditions used to evaluate these constants are as follows:

No-slip condition at the tube wall:

$$V = V_\infty \quad \text{at} \quad R = R_2 \quad \text{or} \quad v = 1 \quad \text{at} \quad r = 1 \tag{7}$$

Shear-stress condition at the bubble surface:

$$-\partial v/\partial R = \sigma_T\,\partial T/\partial Z \quad \text{at} \quad R = R_1 \quad \text{or}$$

$$\partial V/\partial r = [(-\sigma_T A)R_2/\mu V_\infty] \quad \partial\alpha/\partial z \quad \text{at} \quad r = r_1 \tag{8}$$

Heat-flux condition at the bubble surface:

$$\partial T/\partial R = 0 \quad \text{at} \quad R = R_1 \quad \text{or} \quad \partial\phi/\partial r = 0 \quad \text{at} \quad r = r_1 \tag{9}$$

The wall temperature varies linearly along the axial direction. In the chosen coordinate system, it is given by

$$T_w = -AZ + AV_\infty t \Rightarrow f(1) = 0 \tag{10}$$

The mass balance of the liquid results in the following expression:

$$\int_{R_1}^{R_2} 2\pi RV\,\mathrm{d}R = \pi R_2^2 V_\infty \quad \text{or} \quad \int_{r_1}^{1} rv\,\mathrm{d}r = 1/2 \tag{11}$$

where $r_1 = R_1/R_2$

Since the bubble is moving with a constant terminal velocity, the net force acting on it is zero, i.e., the viscous shear force on the bubble surface is balanced by the pressure force on the ends of the bubble. The viscous normal forces on the ends are neglected (valid when $\mu\partial V/\partial Z \ll KL$, i.e., $R_2/R_1 \ll L/R_2$). Thus,

$$-\mu(\partial V/\partial R)_{R=R_1} 2\pi R_1 L = \Delta P\,\pi R_1^2 \Rightarrow K = 2(-\sigma_T A)R_2/(\mu V_\infty r_1) \tag{12}$$

where ΔP is the pressure drop over the length of the bubble.

Equations (7–12) are used to evaluate the six constants in Eqs. (4) and (6). They are

$$C_1 = 0, \; C_2 = (r_1^4 + 1)/(r_1^2 - 1)^2, \; C_3 = -\text{Ma}\,r_1^4/[2(r_1^2 - 1)^2]$$

$$C_4 = -\text{Ma}\,\{(10r_1^4 - 3r_1^2 + 10)/[8(r_1^2 - 1)^2]\}$$

$$K = 8r_1^2/(r_1^2 - 1)^2 \tag{13}$$

If we substitute the value of K from Eq. (13) into Eq. (12), the following expression for the terminal velocity of the bubble is obtained:

$$V_\infty = (-\sigma_T A)R_1\ [(R_2/R_1)^2 - 1]^2/(4\mu) \tag{14}$$

With order-of-magnitude analysis, it may be shown from the results that the radial pressure gradient resulting from liquid acceleration near the ends of the slug is negligible compared to the axial pressure gradient in the liquid film when r_1 is sufficiently close to 1, for both small and large Reynolds numbers. Also, analysis of the normal stress balance at the interface reveals that the surface deformation is negligible if $(-\sigma_T)\ AL/\sigma \ll 1$, i.e., $\Delta\sigma/\sigma$ over the bubble surface is small compared to 1.

Equation (14) shows that the terminal velocity of the gas slug depends on $d\sigma/dT$, the viscosity of the liquid, the imposed temperature gradient, and the slug and tube radii, but is independent of the length of the slug. It is also interesting to note that the terminal velocity does not depend on the thermal diffusivity of the liquid, even though convective terms have been retained in the energy equation. This is because of the fully developed nature of the flow. Some representative values of the terminal velocity of a gas slug of radius 0.4 cm in a tube of radius 0.5 cm and subjected to a temperature gradient of 0.1°C/cm are 0.2 cm/s for liquid nitrogen, 0.07 cm/s for water, and 0.7 cm/s for lithium. The authors are not aware of any published results to compare the predicted values of the terminal velocity of the gas slug.

Conclusions

The steady-state motion of a large gas slug inside a tube filled with liquid and subjected to a linear temperature variation has been analyzed by taking into account the thermally induced gradient of the gas-liquid surface tension. An expression for the terminal velocity of the gas slug has been obtained.

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Convective Heat Transfer with Multiflow in an Annular Pipe of Circular Cross Section

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A LITERATURE survey, including Kakac et al.,[1] Shah and London,[2] Soloukhin and Martynenko,[3] Eckert et al.,[4] Lundberg et al.,[5] Kays and Crawford,[6] Eckert and Drake,[7] Gebhart,[8] Bejan,[9] and Arpaci and Larsen,[10] indicates that convective heat transfer analysis for laminar flows in an annular circular conduit are traditionally based on lumped-parameter studies. The analysis presented herein of convective heat transfer in an annular circular tube subjected to a uniform outside wall temperature is based on actual velocity and temperature fields.

Fully developed laminar flows prevail over most of the flow length in highly compact heat exchangers that employ continuous flow passages geometries. Fully developed laminar flows also prevail in process heat exchangers that have highly viscous liquids as working fluids.

The exact temperature distribution for flows of an annular pipe subjected to a uniform wall heat flux was recently made available by Ebadian et al.[11] Using this distribution as a starting temperature field and employing the successive iteration method presented by Kays and Crawford,[6] Eckert and Drake,[7] Gebhar,[8] Bejan,[9] and Arpaci and Larsen,[10] the temperature fields in both flows are obtained analytically. Calculations were only carried out to the end of the first iteration. This is partly because of the increasing complexity of the calculations. However, even after one cycle of iterations, the values obtained for heat transfer characteristics corresponding to the limiting cases (simple circular pipe flow) are sufficiently close to classical values. For example, the Nusselt number for a circular section obtained after one iteration is 3.7288 compared to the exact value of 3.6568. One recent application of the successive iteration method is described in Ebadian et al.[12] where it was applied to the convective heat transfer problem of a single flow in an elliptic conduit and gave satisfactory results.

It was shown in Ebadian et al.[11] that the dimensionless heat transfer coefficients (Nusselt numbers) on the outer and inner separating surfaces of an annular circular pipe can be defined in three different ways (cases I, II, III, respectively). Since case I of Ebadian et al.[11] is most often applied in practical applications, only this case will be used in these calculations. According to this case, the heat transfer coefficients are based on the mixed flow bulk temperature of the inner and outer flows. The thickness and the thermal resistance of the inner separating wall are also neglected herein. The numerical results are given in graphical form. No experimental work was done and none is reported in the literature.

The importance of this work is in the design of heat exchangers. This information is necessary for designers as well as for the practitioner.

Temperature Fields

By neglecting the thermal resistance of the inner separation wall, and considering the secitons located sufficiently far from both entrances of the conduit where hydrodynamically and thermally fully developed flow conditions prevail, the continuity of temperature distribution is maintained by the forms

$$T_o = CZ + E_o(X,Y), \qquad T_i = CZ + E_{\rm in} + E_i(X,Y) \tag{1}$$

where T_o, T_i and E_o, E_i indicate the temperature and excess temperature distributions in outer and inner regions, respectively, for a constant heat flux condition. $E_{\rm in}$ represents the temperature at the inner separation surface for a constant heat flux case.

The continuity of the two temperature fields to be determined is satisfied by the following forms:

$$\bar{T}_o = \bar{E}_o(X,Y) \qquad \text{and} \qquad \bar{T}_i = \bar{E}_{\rm in} + \bar{E}_i \tag{2}$$

together with the boundary condition $\bar{T}_o = 0$ at the wall.

It must be noted that this selection cannot impose any restriction, and it can be used without any loss of generality of the problem. Therefore, the quantities $\bar{E}_o$, $\bar{E}_i$, and $\bar{E}_{\rm in}$ represent cap temperatures in the outer and inner regions and at the separation surface, respectively.

Received Jan. 14, 1988; revision received April 18, 1987.

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